The Conditional Lucas & Kanade Algorithm

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Contributions

- Establish theoretical connections: **The Lucas & Kanade Algorithm (LK)** ⟷ **Supervised Descent Method (SDM)**
- **The Conditional LK Algorithm**: efficient aligner which achieves comparable performance with little training data

LK

- Aligns a source image $\mathcal{I}$ against a template image $\mathcal{T}$ by minimizing their appearance error

$$\min_{\Delta\mathbf{p}} \|\mathcal{I}(\mathbf{p}) - \mathcal{T}(\Delta\mathbf{p})\|_2^2 \quad \text{(inverse-compositional)}$$

1st-order Taylor approximation

$$\min_{\Delta\mathbf{p}} \left\| \mathcal{I}(\mathbf{p}) - \mathcal{T}(\mathbf{0}) - \nabla\mathcal{T}(\mathbf{0})\frac{\partial\mathcal{W}(\mathbf{x};\mathbf{0})}{\partial\mathbf{p}}\Delta\mathbf{p} \right\|_2^2$$

solve

$$\Delta\mathbf{p} = \underbrace{\boxed{\left(\nabla\mathcal{T}(\mathbf{0})\frac{\partial\mathcal{W}(\mathbf{x};\mathbf{0})}{\partial\mathbf{p}}\right)^{\dagger}}}_{\mathbf{R}} [\mathcal{I}(\mathbf{p}) - \mathcal{T}(\mathbf{0})]$$

SDM

- Learns the appearance-geometry linear relationship from synthetically generated data $\mathcal{S} = \{\Delta\mathbf{p}_n, \mathcal{I}_n(\mathbf{p}_n \circ \Delta\mathbf{p}_n)\}_{n=1}^{N}$
- Linear regressors are trained from independently sampled data per iteration

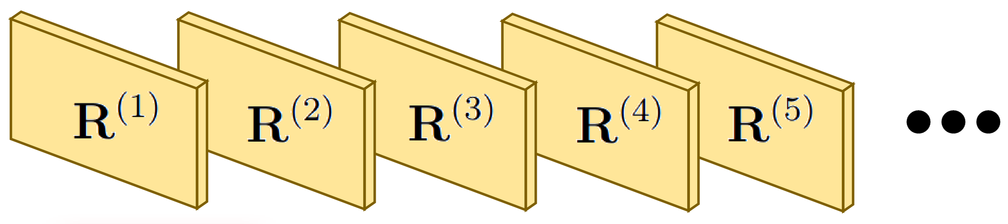
Training: $$\min_{\mathbf{R}} \sum_{n\in\mathcal{S}} \|\Delta\mathbf{p}_n - \mathbf{R}[\mathcal{I}_n(\mathbf{p}_n \circ \Delta\mathbf{p}_n) - \mathcal{T}(\mathbf{0})]\|_2^2 + \Omega(\mathbf{R}) \quad \text{(regularization)}$$

- Prediction of the geometric displacement is learned conditioned on the appearance

Evaluation: $$\Delta\mathbf{p} = \boxed{\mathbf{R}}[\mathcal{I}(\mathbf{p}) - \mathcal{T}(\mathbf{0})]$$

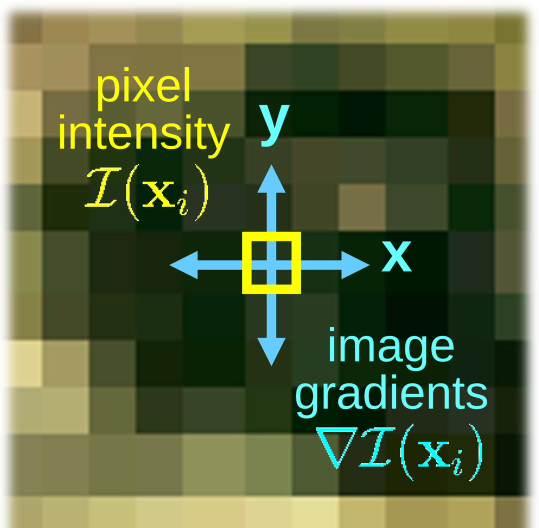
Similarity

- Both assume a linear relationship between appearance and geometry
- Solve for geometric updates iteratively until convergence is reached

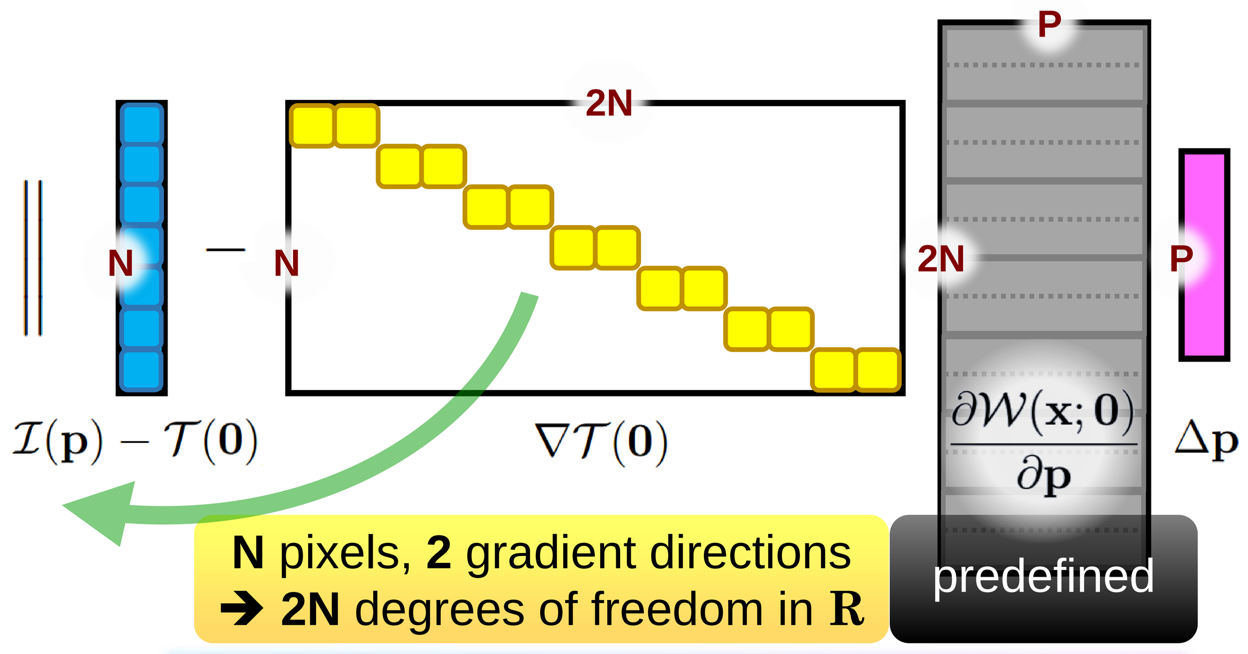


Difference

LK



classically taken via **finite differencing**

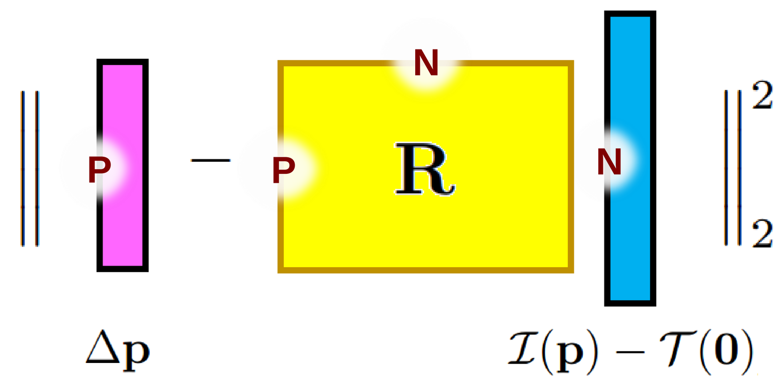


N pixels, **2** gradient directions
➔ **2N** degrees of freedom in $\mathbf{R}$

fits $\Delta\mathbf{p}$ by trying to synthesize $\mathcal{I}(\mathbf{p}) - \mathcal{T}(\mathbf{0})$

- **Pixel independence assumption**
- **Generative appearance synthesis**

SDM



N pixels, **P** warp parameters
➔ **PN** degrees of freedom in $\mathbf{R}$

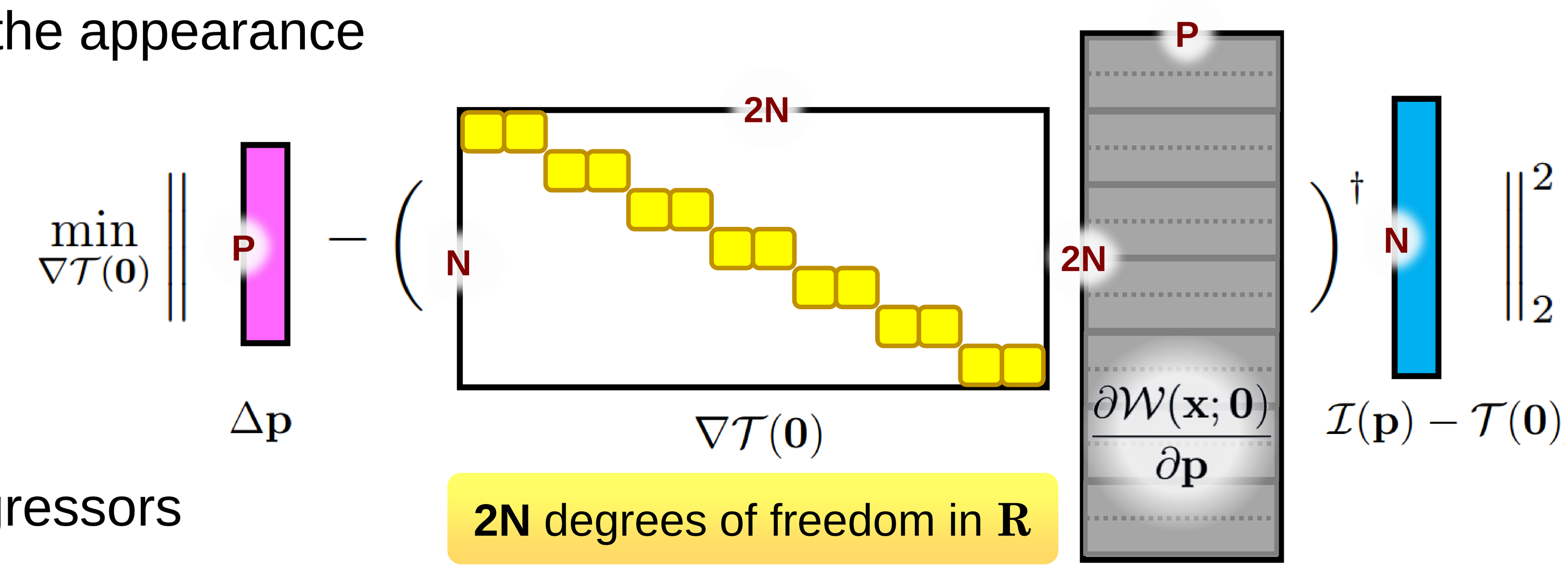
predicts $\Delta\mathbf{p}$ conditioned on $\mathcal{I}(\mathbf{p}) - \mathcal{T}(\mathbf{0})$

- **Full dependency across pixels**
- **Conditional learning objective**

Conditional LK

- Learn the image gradients $\nabla\mathcal{T}(\mathbf{0})$ conditioned on the appearance
- Final regressor: $\mathbf{R} = \left(\nabla\mathcal{T}(\mathbf{0})\frac{\partial\mathcal{W}(\mathbf{x};\mathbf{0})}{\partial\mathbf{p}}\right)^{\dagger}$

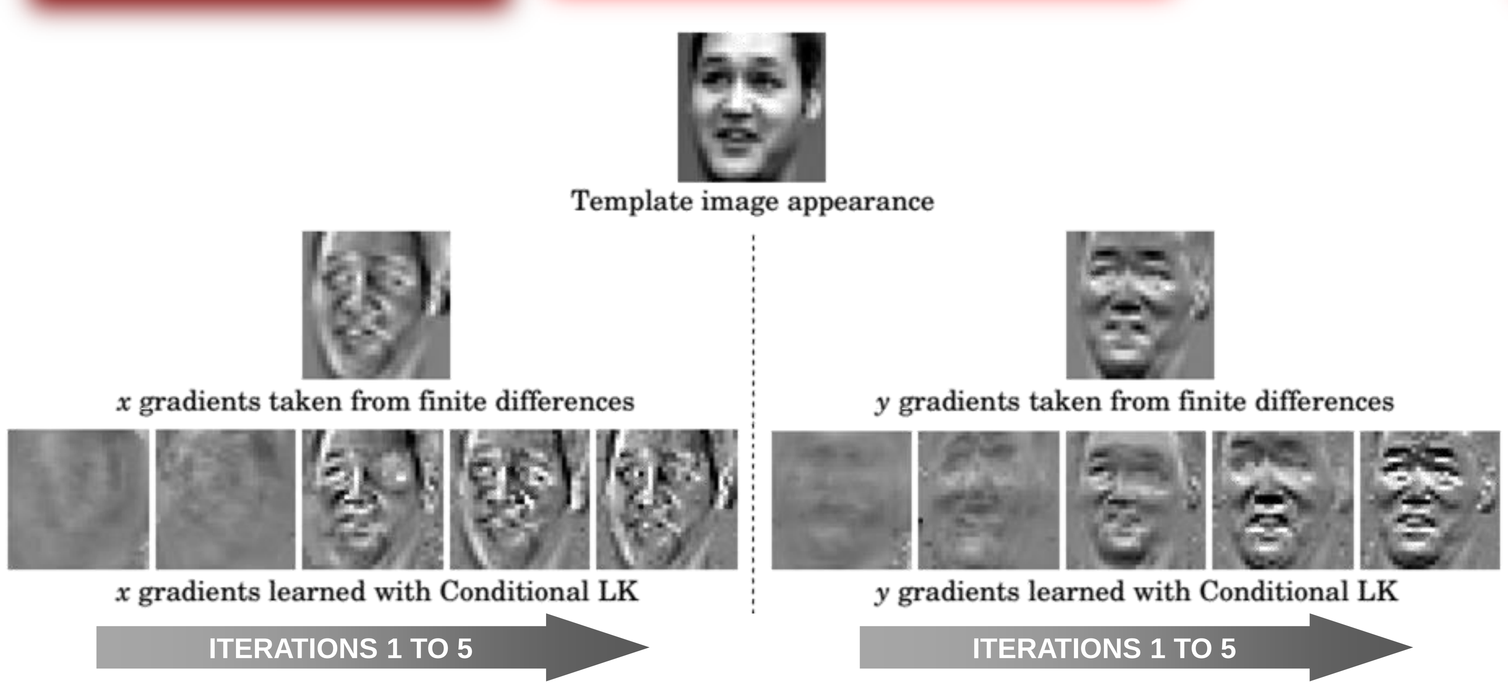
- **Conditional learning objective**
- **Pixel independence assumption**
- **Warp swapping property**:
  Geometric warp functions can be swapped and combined with the conditional image gradients $\nabla\mathcal{T}(\mathbf{0})$ to form another series of linear regressors
- Non-linear least squares problem



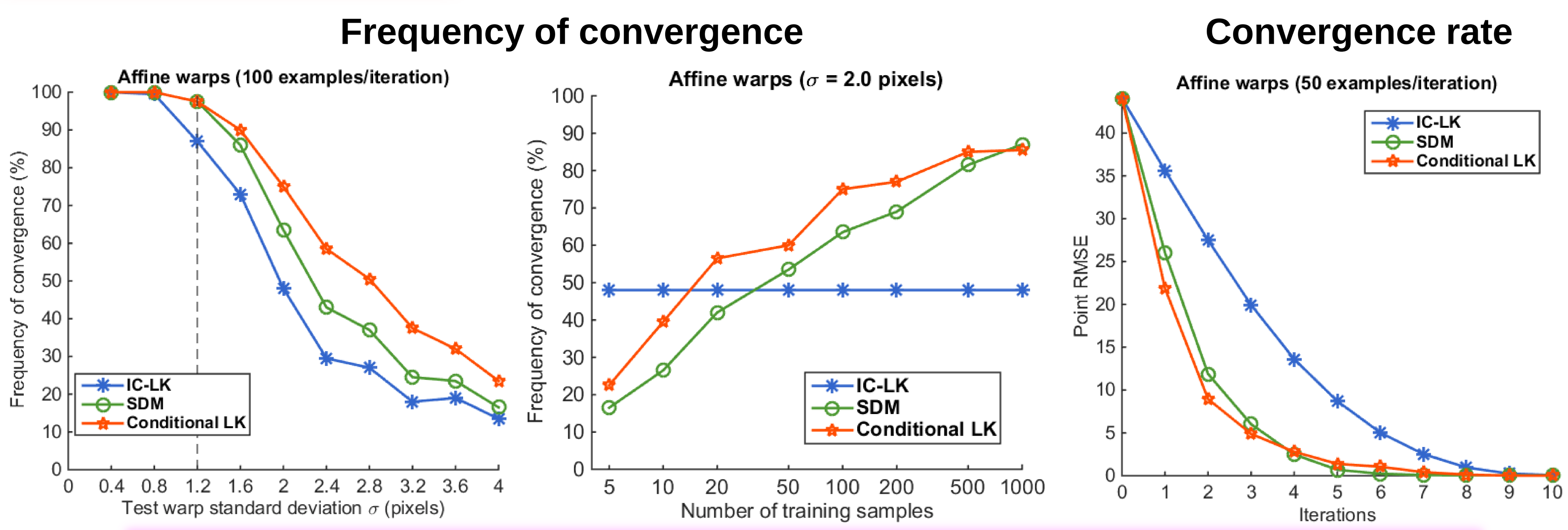
2N degrees of freedom in $\mathbf{R}$

Experiments

Visualization of $\nabla\mathcal{T}(\mathbf{0})$

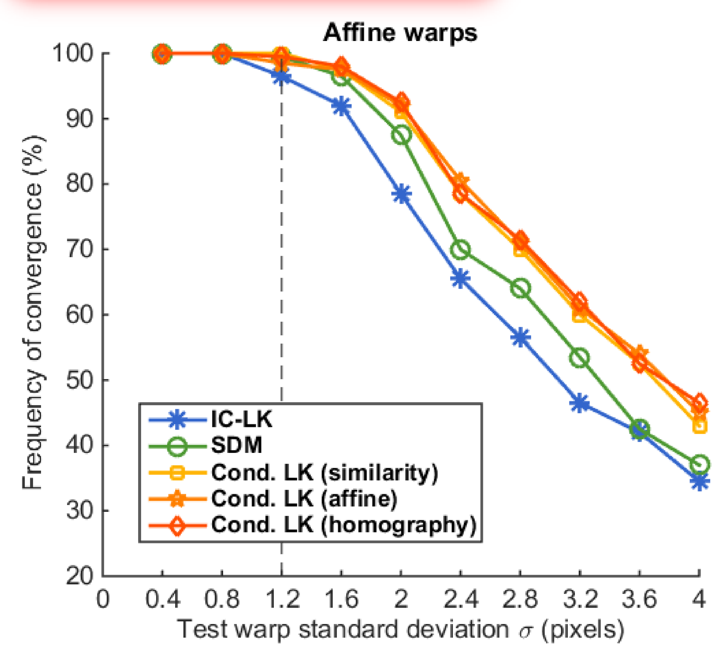


Convergence Analysis



Similar results for different warps, feature images, examples/iteration, and test perturbations

Warp Swapping



Applications

Low frame-rate tracking

BLUE: IC-LK **YELLOW: Conditional LK**